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Several new experimental methods for studying nonlinear behavior in structural dynamics have been developed. Control software for automated bifurcation data acquisition system which can be used to generate high resolution experimental bifurcation diagrams is proving to be useful in identifying critical dynamical regimes for specific systems and to provide an important source of model validation data. A stochastic interrogation method for studying global dynamics and basins of attraction experimentally has been developed and applied. Data obtained with this technique has been used to find basins of attraction, identify global bifurcations, and generate nonlinear models. The same procedure has been used to obtain the first experimental image of homoclinic bifurcation before any stable chaotic solutions are present in the system, and to perform damping measurements in a new way. It has also been shown how spatial correlation measurements and the propoer orthogonal decomposition allow the relationship between spatial and temporal complexity to be examined, and provide "shape functions" which describe the observed dynamics, even when linear normal modes are not well defined. On the theoretical front, a new numerical method for studying nonlinear modal interactions based on the calculation of Lyapunov exponents has also been developed. The method splits the modes into active and passive sets: the active modes interact to contain the attractor, and hence can be considerred "more important" in formulating a low-dimensional model.

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DIMENSIONALITY AND SPATIAL COHERENCE IN THE DYNAMICS OF FLEXIBLE IMPACT OSCILLATORS

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FINAL REPORT

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1. Summary

As a result of the work done for this project, several new experimental methods for studying nonlinear behavior in structural dynamics have been developed. Control software for **an automated bifurcation data acquisition system** which can be used to generate high resolution experimental bifurcation diagrams has been written and applied in a number of cases. More widespread use of such diagrams is proving to be useful in identifying critical dynamical regimes for specific systems, and, more generally, to provide an important source of model validation data. A **stochastic interrogation method for studying global dynamics** and basins of attraction experimentally has been developed and applied. Data obtained with this technique has been used to find basins of attraction, identify global bifurcations, and generate nonlinear models. We have used the same procedure to obtain **the first experimental image of homoclinic bifurcation** (a known precursor to chaos and the cause of chaotic transients) *before* any stable chaotic solutions are present in the system, and to perform damping measurements in a new way. We have shown how spatial correlation measurements and the proper orthogonal decomposition allow **the relationship between spatial and temporal complexity** to be examined, and provide "shape functions", or "modes" which describe the observed dynamics, even when linear normal modes are not well defined. These new experimental tools compliment our simulation and analysis efforts: this gives us a unique capability to compare different methods, and test theoretical predictions. On the theoretical front, we have developed **a new numerical method for studying nonlinear modal interactions which is based on the calculation of Lyapunov exponents** and the definition of the Lyapunov dimension. The method splits the modes into *active* and *passive* sets: the active modes interact to contain the attractor, and hence can be considered "more important" in formulating a low-dimensional model.

2. Statement of Work

An outline of the original work plan is given below. This outline is presented as a reference only: the ultimate scope of the project was greatly expanded from this original plan, as described in Section 3.

I. Experiments

A. Rebuild existing setup: A more rigid mount is needed to reduce unwanted vibrations. Optical probes will be mounted on *two-axis precision adjustable mounts* to give spatial resolution on the order of 0.001".

B. Construct test specimens: Test beams will be steel of various stiffnesses, and at least one fiber-composite specimen. Damping treatments will be applied to allow the effect of damping to be studied. *This is especially important for comparison to numerical modal convergence studies*, since constrained-layer damping can be selectively applied to the beams to effectively control the number of excited modes.

C. Perform baseline tests: These include standard linear frequency response measurements, as well as bifurcation studies to identify areas of interest for further study. For work under this proposal, we plan to *write software to perform automated experimental bifurcation diagrams*: currently these are obtained manually, and are very time-consuming and error prone. The automated bifurcation program will use subroutines and facilities of the Concurrent data acquisition system control program.

D. Take Spatial Correlation Measurements in Chaotic Regimes: In parameter regimes identified in part I.C., we will collect the two-point simultaneous data needed for spatial correlation measurements. In particular, data will be taken to obtain spatial correlation lengths and find proper orthogonal modes in chaotic states. The aim will be to *quantify the degree of spatial complexity* (i.e. the number of degrees of freedom excited) in specific chaotic motions.

E. Collect Data for Correlation Dimension Estimates: This data will be collected simultaneously in order to compare these results to the proper orthogonal mode

dimension estimates. Data will also be collected to track dimensionality and spatial coherence through bifurcations leading up to the given chaotic regime.

II. Experimental Data Analysis

A. **Calculate proper orthogonal modes:** as describe in Section 1, these calculations not only provide spatial information, but also result in an estimate of the number of degrees of freedom involved in the dynamics. We will *compare results from continuous time and impact Poincaré map (discrete time) data* (in theory, these should give the same dimension estimates).

B. **Calculate correlation dimensions:** large data records (10^5 to 10^6 data points) will be used to calculate the correlation dimension via the delay-embedding procedure. Results will be compared to results from part II.A. *This will give an independent confirmation of the correlation dimension estimates*, something which has rarely been done for experimental data.

C. **Correlate Data:** Dimensionality, spatial correlation and modal damping data will be correlated to *determine the relationship between dissipation strength and spatial complexity.*

III. Modeling and Numerical Work

The numerical work will *repeat, with simulations, the work done in items I.C through II.B.* The basic code for simulating impact oscillators has been perfected under current research contracts, however it has only been applied to multi-degree of freedom lumped-parameter models. We will *modify the code to handle Galerkin expansions of continuum models so that modal convergence can be studied.*

IV. Comparison of Simulations and Experiments

The purpose of these comparisons is not merely to validate the theoretical models. *Dimension theory and the proper orthogonal mode technique will be tested* by comparing numerical and experimental bifurcation diagrams: for example, if dimension estimates

indicate that only the first n proper orthogonal modes are needed to describe the dynamics, numerical bifurcation diagrams obtained for fixed n will be obtained to determine whether or not the bifurcation behavior does indeed converge as predicted.

3. Research Accomplishments

All of the main objectives for this project were achieved (see Section 2). In addition, there have been several accomplishments that fall outside of the original scope of this work, but which have occurred as natural outgrowths of the pursuit of the goals listed in the statement of work. The overall goal of the work done for this proposal was to develop methods for studying the effect of system parameters (such as damping and forcing levels) on the *dimensionality* of nonlinear structural systems (i.e., on the number of degrees of freedom needed to model observed dynamics), and to relate this to *spatial complexity* (i.e., to the nature and number of "modes" needed to model the dynamics).

Originally, impact oscillators are a main focus of the work because they seem to be an ideal "test bed" for studying dimensionality and spatial complexity in nonlinear structural dynamics; however, we have not hesitated to explore new experimental systems which appear capable of addressing the same issues. In several cases, our methodological goals were best served by devising different experimental systems. This is as it should be: it must be emphasized that *the methods developed in this work are by no means limited to the study of impacting systems.* We expect that they will find wide application for studying a variety of mechanical and electromechanical systems.

We now summarize the various results in the subsections which follow. (Note: in the interest of brevity, references have been kept to a minimum, and refer to the publications as listed in Section 3.)

Automated Data Acquisition for Bifurcation Analysis

Real-time programs to perform automated experimental bifurcation studies of mechanical and electro-mechanical systems have been written and applied. Despite the commonplace appearance of bifurcation diagrams in the literature, there are still very few detailed experimental bifurcation studies, especially for mechanical systems. We consider this to be an essential tool for our laboratory.

We have developed an experimental apparatus capable of generating bifurcation diagrams of high resolution. A bifurcation diagram is experimentally produced in a straightforward manner by starting with suitable fixed parameter values and waiting until the system being studied reaches a steady state. Data points are then collected in a suitably defined Poincaré section (e.g., once per forcing period for periodically driven systems) so that the steady-state orbit can be determined (current practical methods limit such measurements to stable solutions only). The system's parameter values are then slightly incremented, and the process is repeated until the parameter path of interest is investigated. When high resolution is required, manual control of this process can become very time consuming, tedious, and prone to error, especially for mechanical systems operating at low frequencies and possessing long transient times. This is why most experimental bifurcation diagrams found in the literature have been produced using electronic systems, which operate at much higher frequencies.

We use a workstation-based data acquisition system, and control programs for bifurcation diagrams (and the basin diagrams described in the next section) are written using a library of real-time Fortran subroutines. In addition to the A/D and D/A hardware, the data acquisition system uses a clock board which can be configured with external jumpers to set up the clock cascades needed to synchronize timing and create the burst patterns needed to trigger sampling at the proper frequencies. This clock-based approached to synchronization is natural for periodically forced mechanical systems, since the forcing provides a natural `clock'. However, our data acquisition control programs are quite general, since interrupts can be presented to the hardware as external clock pulses. For example, should position-based Poincaré sections be needed (instead of

stroboscopic sections), simple external triggering circuitry can supply pulses at the appropriate level crossings. The control program has the ability to backtrack in the parameter space at a given point. This feature allows branches which have been reached by jumps to be explored fully. Thus, given sufficient run time, multiple, coexisting solution branches can be traced out, and hysteretic phenomena can be studied.

Experimental bifurcation diagrams have been used successfully to study a driven, two-well oscillator by Cusumano and Kimble (1994, 1995a). Typical runs took 8-16 hours: it was found that certain features of the diagrams can only be found by backtracking, which wd have now included as a standard part of our experimental control algorithm. Possibly the most interesting aspect of the bifurcation diagrams we have found is the large number of jumps between branches and the fact that, though period doubling sequences are observed, the classic period-doubling approach to chaos does not occur. Our experience to date with various electro-mechanical systems suggests that this is the norm. Other applications of the bifurcation data acquisition system can be found in Cusumano and Sharkady (1993, 1995) and Cusumano, Sharkady and Kimble (1994a,b).

Experimental Global Analysis Using Stochastic Interrogation

Beyond questions of local stability, a central role in the study of instability phenomena and the onset of chaos is played by the global structures of phase space, such as basins of attraction, which are associated with the invariant manifolds of unstable orbits. To data, essentially all efforts to visualize and analyze these features have relied on computer simulations: virtually nothing has been done in this area experimentally, especially for mechanical systems. We have developed a new experimental technique that can be used to study global phase space features, such as basins of attraction. The method is based on the idea of *stochastic interrogation*: an ensemble of initial conditions is generated by switching between stochastic and deterministic excitation.

Bifurcation and basin diagrams are conceptually complementary: bifurcation diagrams use steady-state data to explore essentially local phase space phenomena (global phenomena are only hinted

at indirectly); by contrast, basin diagrams use transient data to visualize global phase space structures. This complementarity is reflected in the required control strategy, and necessarily makes collecting basin data a bit more complex, since an ensemble of transients is required, as opposed to a single steady-state.

Basins of attraction are most often represented by two dimensional images in which the initial conditions in each basin are coded with an identifying color. When numerically generating a basin image (or `initial condition map'), one discretizes the desired region into finely spaced grid of initial conditions. Then the trajectory of each initial condition is simulated until it has converged to an attractor. In most physical experiments, however, it is very difficult to specify initial data in this manner. The stochastic interrogation method overcomes this problem by using an interval of random excitation to generate a random initial condition before switching to deterministic forcing. The transient data (i.e. the image of the initial data under the action of the dynamical system) needed to discern asymptotic behavior is collected as soon as the output of the deterministic forcing begins. By repeating this cycle a large number of times, we are able to fill out a portion of the initial condition space near the attractors. Basins of attraction, for example, can be determined by postprocessing the ensemble of orbits to correlate the initial conditions with the appropriate attractor.

Experimental basin data has been used in Cusumano and Kimble (1994, 1995a-c). The transition from simple to complex basin boundaries can be clearly determined with such images. This means that homoclinic bifurcations can be detected even when, as in this case, the only attractors are simple periodic orbits, a significant improvement over previous experimental methods. The ability to detect such basin metamorphoses is important, since the appearance of fractal basin boundaries can greatly change the transient behavior of a system: long chaotic transients can occur after the transition to fractal boundaries, and the appearance of fractal boundaries greatly increases the final state uncertainty, even for very small initial condition uncertainty.

The same method was used to obtain global damping estimates using Liouville's theorem: by defining a *volume decrement* giving the amount of contraction in phase-space volumes in one

forcing periods, the damping coefficients have been obtained and compare closely with those obtained by conventional means for oscillations of the same size. The advantage of this new method is that it is not limited to the linear regime, and thus can be used to give global damping estimates.

In addition to characterizing dynamical quantities such as damping, the stochastic interrogation data can be used to generate nonlinear models of the dyanmical system. We have successfully constructed nonlinear probabilistic models using transition probability matrices. To compute the transition probability matrices, we partition the phase space of our system into M^2 boxes using an $\boldsymbol{M}\times\boldsymbol{M}$ grid. Let B_i be the i^{th} box, and N_i the number of data points contained in it (subscripts range from 1 to M^2). The conditional probability that a point in B_i goes to B_j on the next iterate is

$$P_{ji} = \frac{\mu\left(\phi_T^{-1}(B_j)\cap B_i\right)}{\mu\left(B_i\right)}\,, \tag{1}$$

where μ is the counting measure. Thus, for a given partition, the probability density p_i^n, which gives the probability that the state of the system lies in B_i at iterate n, can be used to determine the probability that the state of the system will lie in B_j at iterate n+1 by

$$p_j^{n+1} = P_{ji}\,p_i^n\,. \tag{2}$$

where summation is implied by the repeated indices. P_{ji} is a transition probability matrix for the system: note that it depends not only on the system parameters, but also on the partition. Using only the first few iterates of the data, we were able to obtain transition probability models which capture significant information concerning the global structure and asymptotic behavior of the system. Current efforts aim at constructing noisy deterministic models using the interrogation data, and on developing hybrid deterministic/probabilistic models using dynamical systems methods and connectionist networks.

Dimensionality, Spatial Coherence, and Modal Interaction

Impact oscillators have proved to be theoretically-important and technologically-useful examples of mechanical, discrete-time nonlinear dynamical systems. Many recent investigations have

examined impacting systems, both from theoretical and experimental points of view (see Cusumano and Bai, 1993, and Cusumano, Sharkady and Kimble, 1994, and the references therein) . However, previous studies of the nonlinear dynamics of impact oscillators have involved systems with, either literally or effectively, one degree of freedom. This is a severe limitation, since flexibility effects should be expected to play an important role in applications. Furthermore, elementary impact theory suggests that many degrees of freedom will be excited by impacts on flexible structures. Thus, flexible impact oscillators appear to be ideal for explorations into the relationship between temporal and spatial complexity in nonlinear structural dynamics.

The experimental system studied by Cusumano, Sharkady and Kimble (1994a,b) is a lightly-damped thin steel beam (0.061 cm x 1.27 cm x 26.52 cm) with a mass of 16 g. The beam is mounted in a horizontal cantilevered position (see Figs. 1 and 2): the cross-sectional axis corresponding to the smallest bending stiffness is oriented vertically, so that the motion is confined to the horizontal plane. The beam is made to vibrate by striking its free end with a sinusoidally vibrating impactor which consists of a 2.2 Kg mass driven by an electromagnetic shaker. The impactor has an axisymmetric, approximately spherical tip, and is positioned such that it is just touching the end of the beam when the system is at rest. Two reflectance compensated fiber-optical displacement probes are used to obtain data from the system. The sensors are mounted on precision adjustable sliders that allow each of the probes to be positioned independently along the length of the beam to an accuracy of 0.025 mm. A third optical probe is used to acquire displacement data from the impactor.

The development of fractal dimension theory and, in particular, of the correlation dimension algorithm, has allowed dimensionality to be studied in physical and numerical experiments on a variety of fluid- and solid-mechanical systems. From our point of view, the utility of fractal dimension estimates lies in the following: given a motion on an attractor with fractal dimension d, the number of phase space dimensions m needed to contain the attractor is bounded by

$$[d] \leq m \leq 2[d]+1, \tag{3}$$

where $[a]$ denotes the next greatest integer to a.

In our experiments, dimension estimates were carried out for three impactor frequencies corresponding to a periodic orbit, a chaotically-modulated period-2 orbit, and a chaotic orbit. At each frequency, 150,000 points of beam tip displacement data were collected, and the time series were delay-reconstruct in the usual manner: let the sampled time series of N points be given by

$$\{x_1, x_2, x_3, \ldots, x_N\}. \tag{4}$$

Then a sequence of m-dimensional delay-reconstructed state vectors,

$$\mathbf{y}^j = (y_1^j, y_2^j, \ldots, y_m^j), \tag{5}$$

can be formed for a fixed delay δ by letting

$$y_i^j = x_{j+\delta(i-1)} \tag{6}$$

for $1 \le i \le m$ and $1 \le j \le N_y$, where $N_y \equiv [N - \delta(m-1)]$. The embedding dimension m must be chosen large enough so that the mapping defined by the delay embedding procedure is invertible: for a *known* phase space dimension n, Taken's theorem states that generic values of δ are guaranteed to give good results as long as $m \geq 2n+1$. However, one typically does not know the true phase space dimension in experiments, and finite precision and noise make careful selection of δ mandatory.

The delay was chosen using the mutual information criterion of: by taking the first minimum of the mutual information, the delayed coordinates are optimal in a specific information-theoretic sense. A mutual information code was run on each data set to determine the time delay δ to use for the embedding. After computing δ, the embedding dimension m was chosen using the method of false nearest neighbors. The basic idea in the false nearest neighbor method is to check how the distances between points in suitably defined neighborhoods change as the embedding dimension is increased. Points which appear to be close to each other in dimension m but which become far apart in dimension m+1 are defined to be false nearest neighbors. Using the value of m resulting from the false nearest neighbors algorithm (call it m_F), together with the previously determined value of δ, the data sets were then delay-reconstructed, and the correlation dimension d_C was calculated. Note that the values of d_C and m can be checked for internal consistency from inequality (3). As a final check, d_C was computed for a few values of $m > m_F$ and in all cases was found to be in good agreement with the value obtained with $m = m_F$.

While dimension theory is useful, it is only of limited use for system modelers: fractal dimension estimates give bounds on the number of degrees of freedom, but they do not indicate appropriate configuration variables to use in formulating models of the system. In particular, fractal dimensions do not yield any spatial information concerning important "shape functions" or "modes" in the system, information which is usually of great significance in engineering applications. One method for studying dimensionality that addresses some of these limitations is based on the proper orthogonal decomposition of the covariance of a random function, applied originally in mechanics to the study of turbulent fluid flows. The object of study in the proper orthogonal decomposition (abbreviated POD) method is the spatial correlation given by the dyadic product:

$$\mathbf{R}(s,s') = \langle x(s,t)x(s',t)\rangle, \tag{7}$$

where $\boldsymbol{x}$ is an appropriate field variable in space s and time t.

In the work of Cusumano and Bai (1993) and Cusumano, Sharkady and Kimble (1994a,b), the spatial correlation is the tensor given by:

$$\begin{aligned}\mathbf{R} &= \langle \mathbf{x}(t)\mathbf{x}(t)\rangle = \langle x_i(t)x_j(t)\hat{\mathbf{e}}_i\hat{\mathbf{e}}_j\rangle \\ &= \langle x_i(t)x_j(t)\rangle \hat{\mathbf{e}}_i\hat{\mathbf{e}}_j \equiv R_{ij}\hat{\mathbf{e}}_i\hat{\mathbf{e}}_j,\end{aligned} \tag{8}$$

where $x_i(t)$ is the displacement at the i^{th} point along the beam. Clearly, $\mathbf{R}$ is a Hermitian tensor, and thus its eigenvectors form a complete orthogonal set. Denoting the eigenvectors of $\mathbf{R}$ (called *proper orthogonal modes*, or POM) by $\hat{\boldsymbol{\phi}}_i$ and expanding the configuration vector $\mathbf{x}(t)$ as $\mathbf{x}(t) = \sum_i \alpha_i(t)\hat{\boldsymbol{\phi}}_i$ one has $\mathbf{R} = \langle \alpha_i(t)\alpha_j(t)\rangle \hat{\boldsymbol{\phi}}_i\hat{\boldsymbol{\phi}}_j$ Since the $\hat{\boldsymbol{\phi}}_i$ are eigenvectors of $\mathbf{R}$, this implies that

$$\hat{\boldsymbol{\phi}}_i\cdot\mathbf{R}\hat{\boldsymbol{\phi}}_j = \langle \alpha_i(t)\alpha_j(t)\rangle = \hat{\boldsymbol{\phi}}_i\cdot\lambda_i\hat{\boldsymbol{\phi}}_j = \lambda_i\delta_{ij}. \tag{9}$$

Equation (5) demonstrates that the modal amplitudes $\alpha_i(t)$ are linearly uncorrelated, and the associated eigenvalues are simply the mean-square proper orthogonal modal amplitudes (it also shows that $\mathbf{R}$ is positive definite). In fact, it can be shown that the eigenvectors of $\mathbf{R}$ (i.e., the POM) give mode shapes that are optimal in a least squares sense: they capture more power per mode than any other set of basis functions, a result also known as the Karhunen-Loève decomposition. The sum of the λ_i is equal to the total mean square amplitude (the "power") of the response at the measured points, and approaches the true mean square amplitude as the number of measured points increases. Thus, the POD method furnishes an alternative measure of dimensionality: the number

of degrees of freedom needed to capture the observed dynamics can be estimated as the number of POM needed to exceed a fixed fraction (say 99%) of the total power.

Finally, on the theoretical front, we have developed a new statistical method for studying nonlinear modal interactions which is based on the calculation of Lyapunov exponents and the definition of the Lyapunov dimension (Lin and Cusumano, 1994, and Cusumano and Lin, 1995). The method computes the correlations between components (with respect to a suitable set of modes) of the Lyapunov vectors by averaging over time along the orbit. The technique identifies statistically-invariant subspaces with the stability properties of the associated Lyapunov exponent, and can be used both to study steady-state behavior, or, if local Lyapunov exponents are used, transient events. The method splits the modes into *active* and *passive* sets: the active modes interact to contain the attractor (and hence are "more important"), whereas the passive modes serve only as power sources or sinks to the active modes. This has important implications for the derivation of low-dimensional models of continuous systems. Perhaps most importantly, we believe that this method is translatable to physical experiments.

Conclusions

The stochastic interrogation method described here generates data sets which allow for the experimental visualization and analysis of global features in the phase space of nonlinear oscillators. We have demonstrated the usefulness of this approach by applying it to the study of a driven, two-well nonlinear magneto-mechanical oscillator. The resulting data yielded images of basins of attraction for the system, visualization of the flow of initial states on the Poincaré section, and an estimate of the linear damping coefficient using a novel approach based on Liouville's Theorem.

While impact oscillators are technologically relevant objects of study in their own right (as, for example, common sources of noise in machinery), our main interest here has been in using them as an experimentally tractable model problem for studying spatio-temporal interaction in structural systems. The bifurcation diagrams suggest that the typical responses of such systems can be expected to be chaotic, and the power spectra typically show excitation in bandwidths spanning tens

of linear modes: hence flexible impact oscillators present themselves as a potentially useful paradigm for solid mechanical "turbulence". Nevertheless, in all of the cases studied here, over 90% of the mean square response amplitude was shown to be captured by the 1st proper orthogonal mode (POM).

From a general, methodological perspective, the importance of comparing several different methods of experimental data analysis on complex signals has been demonstrated. By combining topological methods, like dimension estimates and false nearest neighbor analysis, with the temporal analysis of the mutual information calculation and the spatial analysis provided by the proper orthogonal decomposition, subtle distinctions can be made between different stationary states, even those with similar dimensionality. Statements concerning the "strength" of chaos in a given system must be made carefully: different measures can give quite different answers to such questions.

We have demonstrated the utility of the proper orthogonal decomposition method for structural vibration problems. The deliberate excitation of chaotic impacting responses may even be of practical use for engineering modal analysis. This is especially true since the proper orthogonal decomposition is more general than standard modal analysis methods, since it makes no reference to linearity: being entirely based on statistics, it will yield useful information even when the response between impacts is nonlinear.

A new statistical technique which identifies interacting modes and the average stability properties of the associated subspaces has been developed. The technique employs the Lyapunov vectors used in the calculation of the Lyapunov exponents. We have shown how this method can be used to split the modes in system into active and passive sets: active modes interact to contain the attractor, whereas passive modes behave like isolated driven oscillators.

We have presented a generally-applicable method for studying modal interactions. The method computes the correlations between components (with respect to a suitable set of modes) of the Lyapunov vectors by averaging over time along the orbit. The technique, in effect, identifies statistically-invariant subspaces with the stability properties of the associated Lyapunov exponent,

For the analysis of steady-state motions, the technique offers several advantages over direct use of the modal amplitudes to compute the required correlations. First, not only are interacting modes identified, but the average stability properties associated with the modes are found. Second, via the definition of the Lyapunov dimension, the method provides an objective way of splitting the modes into active and passive subspaces. Thus a practical limitation of dimension theory is overcome: previously, fractal dimension estimates could be used to estimate the number of modes required, but no information concerning the best modes to use was available. In addition, we have shown with this method that modal selection based on power estimates (i.e., the magnitude of the modal variances) may not be as physically meaningful as selection based on the active-passive dichotomy defined by the Lyapunov dimension.

These results suggest a conceptual framework in which to formulate reduced-order models of systems with many degrees of freedom. Using a technique such as the one presented here, active and passive modes should be identified. The reduced system need only include the active modes, however positive and negative dissipation terms must be added to the model to account for power flow from and to the passive modes. Further efforts are needed to determine the best way to formulate these dissipation terms.

The methods described above continue to be refined and applied to a variety of structural systems. Ultimately it is hoped that these techniques from dynamical systems theory will allow us to ask fundamental physical questions regarding the nature of dissipation in nonlinear structural responses, and its relation to spatial complexity. Such information will play an important role in the design of lighter, quieter, and more reliable mechanical systems.

3. Publications

The following publications have been accomplished as part of this grant.

Articles (refereed journals, proceedings and book chapters)

J. P. Cusumano, "Spatial coherence in the chaotic dynamics of multi-degree-of-freedom impact oscillators." *Proceedings of the ASCE Engineering Mechanics Specialty Conference: Mechanics Computing in the 90's and Beyond*, Columbus, OH, May 20-22, 1991.

J. P Cusumano, "The Experimental Study of Bifurcation, Dimensionality and Global Dynamics in Nonlinear Mechanical Systems", to appear in *Nonlinear Dynamics of Structures*, D. Inman and A. Guran, eds., 1995.

J. P. Cusumano, "Experimental Observation of Nonlinear Modal Interaction in a Kicked Flexible Oscillator", in preparation, 1995.

J. P. Cusumano and B.-Y. Bai, "Period-infinity Periodic Motions, Chaos, and Spatial Coherence in a 10 Degree of Freedom Impact Oscillator", *Chaos, Solitons and Fractals*, vol. 3 no. 5, pp. 515-536, 1993.

J. P. Cusumano and B. W. Kimble, "Experimental Observation of Basins of Attraction and Homoclinic Bifurcation in a Magneto-Mechanical Oscillator", in *Nonlinearity and Chaos in Engineering Dynamics*, J. M. T. Thompson and S. Bishop, eds., Wiley, New York, 1994.

J. P. Cusumano and B. W. Kimble, "A Stochastic Interrogation Method for Experimental Measurements of Global Dynamics and Basin Evolution: Application to a Two-well Oscillator", *Nonlinear Dynamics*, in press, 1995a.

J. P. Cusumano and B. W. Kimble, "Experimental Observation of a Strange Nonattracting Set", in preparation, 1995b

J. P. Cusumano and B. W. Kimble, "Theoretical and Numerical Validation of the Stochastic Interrogation Experimental Method", in preparation, 1995c.

J. P. Cusumano and D.-C. Lin, "Bifurcation and Modal Interaction in a Simplified Model of Bending-Torsion Vibrations of the Thin Elastica", *ASME Journal of Vibrations and Acoustics*, vol. 117, no. 1, pp. 30-42, 1995.

J. P. Cusumano and M. T. Sharkady, "An Experimental Study of Bifurcation and Chaos in a Dynamically-Buckled Magnetoelastic System", in *Dynamics and Vibration of Time-Varying Systems and Structures*, DE vol. 56, S. C. Sinha and R. H. Evan-Iwanoski, eds., the 14th ASME Biennial Conference on Mechanical Vibration and Noise, Albuquerque, NM, September 19-22, American Society of Mechanical Engineers, New York, 1993.

J. P. Cusumano and M. T. Sharkady, "An Experimental Study of Bifurcation, Chaos, and Dimensionality in a System Forced Through a Bifurcation Parameter", *Nonlinear Dynamics*, in press, 1995.

J. P. Cusumano, M. T. Sharkady, and B. W. Kimble, "Spatial Coherence Measurements of a Chaotic Flexible-Beam Impact Oscillator", in *Aerospace Structures: Nonlinear Dynamics and System Response*, J. P. Cusumano, C. Pierre, S. Wu, eds., AD vol. 33, the 1993 ASME WAM, New Orleans, 11/28-12/3, American Society of Mechanical Engineers, New York, 1994a.

J. P. Cusumano, M. T. Sharkady, and B. W. Kimble, "Experimental Measurements of Dimensionality and Spatial Coherence in the Dynamics of a Flexible-beam Impact Oscillator", *Philosophical Transactions of the Royal Society*, vol. 347, pp. 421-438, 1994b.

D.-C. Lin and J. P. Cusumano, "Analysis of Modal Interactions in Chaotic Bending-Torsion Vibrations Using the Eigenvectors of the Oseledec Matrix", the *Proceedings of the 1993 Asia-Pacific Vibration Conference*, November 14-18, 1993, Kitakyushu, Japan, 1993.

Books

Aerospace Structures: Nonlinear Dynamics and System Response, J. P. Cusumano, C. Pierre, S. T. Wu, eds., AD vol. 33, American Society of Mechanical Engineers, New York, 1993.

4. Participating Personnel

Graduate Students

Bai, B.-Y., MS in Engineering Mechanics, "The Study and Simulation of a 10 Degree-of-Freedom Impact Oscillator," December, 1991.

Lin, D.-C., PhD in Engineering Mechanics, "Stability, Modal Couplings, and Local Bifurcations in Bending-Torsion Forced Vibrations of a 3-D Elastic Rod," August, 1992.

Bump, T. A., MS in Engineering Mechanics, "Continuation of Periodic Solutions with Applications to Problems in Nonlinear Vibrations," August, 1992.

Lindsley, N. J., MS in Engineering Mechanics, "A Finite-Strain Ring Model for Pneumatic Tires," May, 1993.

Kimble, B. W., MS in Engineering Mechanics, "Application of Experimental Methods for Producing Bifurcation Diagrams and Basins of Attraction to a Two-Well Potential Mechanical System," May, 1993.

Atz, C. B., MS in Engineering Mechanics, "The Evaluation of Lyapunov Exponents from Experimental Signals", May, 1993

Sharkady, M. T., MS in Engineering Mechanics, "An Experimental Study of a Flexible Beam Impact Oscillator", August, 1994.

Undergraduate Students

Sharkady, M. T., BS Honors Thesis in Engineering Science, "Experimental Investigation of a Magneto-Mechanical Oscillator," May, 1992.

Cornwell, J., BS Honors Thesis in Engineering Science, "An Investigation into the Dynamics of a Flexible Beam Oscillator Perpetuated by Magnetic Kicks,", May, 1992.

Bennett, A., BS Honors Thesis in Engineering Science, "The Dynamics of Coupled Double Scroll Oscillators," expected May, 1993.

5. Presentations

(All presentations by J. P. Cusumano unless otherwise noted)

"Dimensionality and Spatial Coherence in the Dynamics of Flexible Impact Oscillators," AFOSR Contractor's Meeting on Structural Dynamics, October 9-11, 1991, Dayton, Ohio. (Invited Lecture)

"Low-Dimensional Chaos in a Flexible Tube Conveying Fluid," Winter Annual Meeting of the ASME, Atlanta, December 1-6, 1992. (With M. P. Païdoussis and S. G. Copeland)

"An Experimental and Numerical Investigation into the Dynamics of a System Forced Through a Bifurcation Parameter," Department of Aerospace Engineering Colloquium, University of Illinois, Urbana-Champagne, January 24, 1992. (Invited Lecture).

"The Dynamics of Impact Oscillators," Department Seminar, Engineering Science & Mechanics, Penn State University, March 1992 (Invited Lecture)

"Analysis of a magneto-Mechanical Oscillator," The 6th National Conference on Undergraduate Education, University of Minnesota, March, 1992, (Presented by M. T. Sharkady)

"An Experimental Study of a System Forced Through a Bifurcation Parameter," ASME Summer Annual Meeting, Phoenix, Arizona, April, 1992 (Invited Lecture)

"The Dynamics of a System Forced Through a Bifurcation Parameter: A Case Study of Nonlinear Dynamics at the Undergraduate Level." ASEE Annual Conference, Toledo, Ohio, June 21-25, 1992 (with M. T. Sharkady; Invited Lecture).

"Spatial Coherence and Bifurcation in the Dynamics of Flexible Impact Oscillators," Department of Mechanical and Aerospace Engineering Colloquium, Case Western Reserve University, Cleveland, Ohio, October, 1992 (Invited Lecture).

6. Interaction with other Laboratories and Agencies

Organization:	Vehicle Subsystems Division; WL/FIV; WPAFB, OH 45433.
Contact:	Dr. Arnold Mayer, Chief Scientist
Activity:	MS thesis advisor for N. J. Lindsley (see section 4), an employee at WL/FIV, and informal consultations with Dr. Mayer. Problem deals with high-speed tire instability. Developed continuum mechanics model for thesis, and discussed nonlinear dynamics applications from theoretical and experimental points of view. This relationship is ongoing.